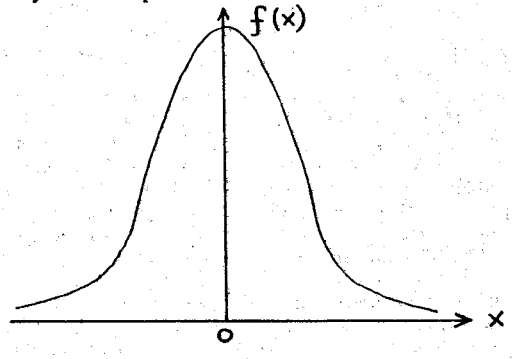
## HP-15C NORMAL DISTRIBUTION

by David Hodges (347)

The normal distribution is the most common continuous distribution in statistics. It is used to model a whole range of real life situations from the life of motor car tyres to the spread of data from a scientific experiment.

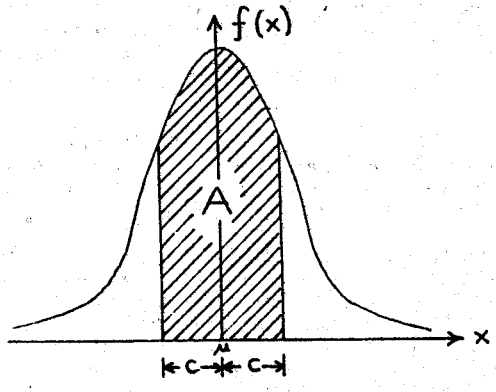
With the x-axis scaled in standard deviations, the familiar bell shaped curve is defined by the expression:-




$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

Whilst the density function, f(x), is easy to evaluate on a calculator or computer, many problems require part of the area under the curve to be determined. This is difficult because the expression for f(x) is impossible to integrate analytically, although it can be shown that the total area under the curve is 1. Calculating an x value corresponding to a given area under part of the curve is even more of a problem.

Fortunately, modern calculators like the HP-15C, HP-34C and HP-41 with Advantage ROM provide SOLVE and INTEGRATE functions which allow the computations to be performed easily, and if desired to a much greater accuracy than is usually found in statistical tables. Although the program given here is for the HP-15C, it can easily be adapted to suit any of the other machines.




Three functions are provided. The first computes the normal density function, f(x), for a given x. The second computes the area A (0 <= A < 1) under the curve captured by the confidence interval +/- c standard deviations either side of the distributon mean μ (ie. zero). The diagram above makes this more clear. Finally, the third function computes c given A, and is an example of how SOLVE and INTEGRATE may be used together.

### User Instructions

x [f][A] ----> f(x)

c [f][B] ----> A

A [f][C] ----> c

The calculation time of [B] and [C] - especially the latter - can take several minutes, depending on the display mode of the HP-15C. The manual for the calculator contains more information on how the display mode affects the accuracy of the final result when using SOLVE and INTEGRATE.

### Example

Calculate: (1) the value of f(x) when x=1 standard deviation; (2) the proportion of the area under the curve captured by the limits x=-1 and x=+1 standard deviations (ie. c=1) (total area under curve = 1); and (3) the confidence interval corresponding to 95% of the area under the curve (ie. find c when A=0.95).

| INPUT | KEYS | OUTPUT | |
|---|---|---|---|
| | f USER<br>f SCI 2 | | Set USER and SCI 2 modes. |
| 1 | A | 2.42 -01 | f(x)=0.242 |
| 1 | B | 6.83 -01 | 68.3% of the area lies between +/- 1 standard devns. |
| 0.95 | C | 1.96 00 | +/- 1.96 standard devns captures 95% of the area under the curve. |

### Program Listing

```
f LBL C     001-42,21,13    f LBL 0     023-42,21, 0
GSB 2       002-    32  2   ENTER       024-       36
x           003-       20   CHS         025-       16
STO I       004-    44 25   x<>y        026-       34
1           005-        1   f ∫y   1    027-42,20, 1
ENTER       006-       36   RCL - I     028-45,30,25
2           007-        2   g RTN       029-    43 32
f SOLVE 0   008-42,10, 0
g RTN       009-    43 32   f LBL 1     030-42,21, 1
                            g x²        031-    43 11
f LBL B     010-42,21,12    2           032-        2
0           011-        0   /           033-       10
STO I       012-    44 25   CHS         034-       16
R↓          013-       33   e^x         035-       12
GSB 0       014-    32  0   g RTN       036-    43 32
GSB 2       015-    32  2
/           016-       10   f LBL 2     037-42,21, 2
g RTN       017-    43 32   g π         038-    43 26
                            ENTER       039-       36
f LBL A     018-42,21,11    +           040-       40
GSB 1       019-    32  1   √x          041-       11
GSB 2       020-    32  2   g RTN       042-    43 32
/           021-       10
g RTN       022-    43 32
```